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Aesthetic Fields: A Topological Particle

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We extend the complex aesthetic field equations for A^i_{jk} and B^i_{jk} so as to have symmetry between these fields. We find computer solutions in which the confluence region takes on a loop shape. We call such a particle a "topological" particle.

1. INTRODUCTION

In our previous paper (Muraskin 1981) we obtained an extended particle system in four dimensions within complex aesthetic field theory. Large negative numbers were found in close proximity to large positive numbers. The magnitude of the fields grew very large in this "confluence" region. These large magnitudes persisted in time. Although we made no effort to find what the values of the fields were in this confluence region, we can say that the values remained large compared to the surrounding region (several orders of magnitude at least, with the possibility of many orders of magnitude). This effect we shall term *nonattenuation*. The particle system was bounded by regions in which the fields were small in magnitude. All components of the field had large magnitudes in the confluence region. We never saw a large magnitude maximum or minimum appearing by itself. The regions of large positive numbers and large negative numbers were always close by to one another. Such an effect we shall term *confinement*. A limitation with our results was that we observed the confluence region to extend in z more and more for high $|t|$ ($\Delta t \sim 10^2$). On the other hand, the spread in x and y for any particular z was quite small.

In this paper we extend the equations for A^i_{jk} and B^i_{jk}. We shall find a solution in which the confluence region lies (in a course sense) on a closed curve. We shall call such a particle a *topological* particle.

2. NEW EQUATIONS FOR A^i_{jk}, B^i_{jk}

In Muraskin (1980) we introduced complex fields into aesthetic field theory. We took

$$\Gamma^i_{jk} \equiv A^i_{jk} + iB^i_{jk} \tag{1}$$

with A^i_{jk}, B^i_{jk} real. The field equations,

$$\frac{\partial \Gamma^i_{jk}}{\partial x^l} = \Gamma^i_{mk}\Gamma^m_{jl} + \Gamma^i_{jm}\Gamma^m_{kl} - \Gamma^m_{jk}\Gamma^i_{ml} \tag{2}$$

then lead to the following equations for A^i_{jk} and B^i_{jk}:

$$\frac{\partial A^i_{jk}}{\partial x^l} = A^i_{mk}A^m_{jl} - B^i_{mk}B^m_{jl} + A^i_{jm}A^m_{kl} - B^i_{jm}B^m_{kl} - A^m_{jk}A^i_{ml} + B^m_{jk}B^i_{ml} \tag{3}$$

$$\frac{\partial B^i_{jk}}{\partial x^l} = A^i_{mk}B^m_{jl} + B^i_{mk}A^m_{jl} + A^i_{jm}B^m_{kl} + B^i_{jm}A^m_{kl} - A^m_{jk}B^i_{ml} - B^m_{jk}A^i_{ml} \tag{4}$$

Below we shall discuss an extension of these equations.

In aesthetic field theory all tensors are treated in a uniform manner so far as their change is concerned. That is we have for a vector

$$dA_i = \Gamma^j_{ik} A_j \, dx'^k \tag{5}$$

A second-rank tensor behaves like the product of two vectors

$$dg_{ij} = \left\{ \Gamma^t_{ik} g_{tj} + \Gamma^t_{jk} g_{it} \right\} dx'^k \tag{6}$$

A third-rank tensor behaves like the product of three vectors

$$d\Gamma^i_{jk} = \left\{ \Gamma^i_{mk}\Gamma^m_{jl} + \Gamma^i_{jm}\Gamma^m_{kl} - \Gamma^m_{jk}\Gamma^i_{ml} \right\} dx'^l \tag{7}$$

This aesthetic principle is maintained whether or not the fields are complex. Complex fields are a mathematical way to deal with two real fields [in (3) and (4) all quantities are real] such that the aesthetic idea of treating all tensors uniformly, with respect to their change, is maintained.

We can go one step further. All tensors are treated in a uniform way even if the dx'^k is complex as well. By choosing

$$dx'^k = dx^k - i\,dx^k \tag{8}$$

with dx^k real, we obtain

$$\begin{aligned}\frac{\partial A^i_{jk}}{\partial x^l} &= A^i_{mk}A^m_{jl} - B^i_{mk}B^m_{jl} + A^i_{mk}B^m_{jl} + B^i_{mk}A^m_{jl}\\ &\quad + A^i_{jm}A^m_{kl} - B^i_{jm}B^m_{kl} + A^i_{jm}B^m_{kl} + B^i_{jm}A^m_{kl}\\ &\quad - A^m_{jk}A^i_{ml} + B^m_{jk}B^i_{ml} - A^m_{jk}B^i_{ml} - B^m_{jk}A^i_{ml}\end{aligned} \tag{9}$$

and

$$\begin{aligned}\frac{\partial B^i_{jk}}{\partial x^l} &= A^i_{mk}B^m_{jl} + B^i_{mk}A^m_{jl} - A^i_{mk}A^m_{jl} + B^i_{mk}B^m_{jl}\\ &\quad + A^i_{jm}B^m_{kl} + B^i_{jm}A^m_{kl} - A^i_{jm}A^m_{kl} + B^i_{jm}B^m_{kl}\\ &\quad - A^m_{jk}B^i_{ml} - B^m_{jk}A^i_{ml} + A^m_{jk}A^i_{ml} - B^m_{jk}B^i_{ml}\end{aligned} \tag{10}$$

These equations have the property that under the transformation

$$\begin{aligned}A^i_{jk} &\to B^i_{jk}\\ B^i_{jk} &\to A^i_{jk}\end{aligned} \tag{11}$$

Equation (9) goes into equation (10) and equation (10) goes into equation (9). This is desirable, since the initial data for A^i_{jk} are not to be favored in any way over B^i_{jk}. Otherwise it would not be clear which set of data we should take for A^i_{jk} and which set for B^i_{jk}. What we are doing then is treating A^i_{jk} and B^i_{jk} in a similar fashion. Note that the right-hand sides of (9) and (10) are not equal to one another and are not the negatives of one another. Another set of equations we shall consider is

$$\begin{aligned}\frac{\partial A^i_{jk}}{\partial x^l} &= A^i_{mk}A^m_{jl} - B^i_{mk}B^m_{jl} - A^i_{mk}B^m_{jl} - B^i_{mk}A^m_{jl}\\ &\quad + A^i_{jm}A^m_{kl} - B^i_{jm}B^m_{kl} - A^i_{jm}B^m_{kl} - B^i_{jm}A^m_{kl}\\ &\quad - A^m_{jk}A^i_{ml} + B^m_{jk}B^i_{ml} + A^m_{jk}B^i_{ml} + B^m_{jk}A^i_{ml}\end{aligned} \tag{12}$$

and

$$\frac{\partial B^i_{jk}}{\partial x^l} = A^i_{mk}B^m_{jl} + B^i_{mk}A^m_{jl} + A^i_{mk}A^m_{jl} - B^i_{mk}B^m_{jl}$$

$$+ A^i_{jm}B^m_{kl} + B^i_{jm}A^m_{kl} + A^i_{jm}A^m_{kl} - B^i_{jm}B^m_{kl}$$

$$- A^m_{jk}B^i_{ml} - B^m_{jk}A^i_{ml} - A^m_{jk}A^i_{ml} + B^m_{jk}B^i_{ml} \qquad (13)$$

These equations can be obtained by choosing dx'^k to be

$$dx'^k = dx^k + i\,dx^k \qquad (14)$$

with dx^k real. Here under the transformation

$$A^i_{jk} \to -B^i_{jk}$$

$$B^i_{jk} \to -A^i_{jk} \qquad (15)$$

equation (12) goes into equation (13) and equation (13) goes into (12).

The new sets of equations (9) and (10) and (12) and (13) are integrable once the integrability equations for (3) and (4) are satisfied. The integrability equations for (3) and (4) were discussed in Muraskin (1980). As a check we observed on the computer that going along different paths gives the same answers for fields to high accuracy (12 decimal places in sample runs). Integrability is maintained at all points provided it is satisfied at the origin point as in our previous work.

In detail, (16) has the structure, when expanding (7) using (1) and (8) or (14),

$$dA^i_{jk} + i\,dB^i_{jk} = \{(\mathrm{Re}) + i(Im)\}\,dx^l \qquad (16)$$

$\partial A^i_{jk}/\partial x^l$ is identified with (Re) and $\partial B^i_{jk}/\partial x^l$ is identified with (Im), where $\partial/\partial x^l$ is real. All the terms in (9) and (10) and (12) and (13) are real.

Thus (8) and (14) can be looked upon as a mathematical device that preserves the aesthetic features of the theory and leads to symmetry between A^i_{jk} and B^i_{jk}.

There are now eight coordinates associated with a point since space-time is complex. But only four of these coordinates are arbitrary (as we are normally accustomed to). The other four are fixed. We have taken them to be equal (up to sign) with the usual four coordinates.

3. A TOPOLOGICAL PARTICLE

We use the same origin point data as in Muraskin (1981). These origin point data have the group theoretical properties described there.

For equations (9) and (10) we did not find any confluence structure in the work we did. The maps looked pretty much similar to those maps described in Muraskin and Ring (1975). We mapped up to $z=10$.

On the other hand (12) and (13) led to a "topological" particle. Our ensuing discussion will be restricted to the set (12) and (13).

In Figure 1 we see a map at $t=0, z=0$, with a 0.2 grid for A^1_{11} (a representative component). Already we see more structure than we have seen previously in aesthetic field theory (provided the integrability equations are satisfied). There are eight planar maxima and minima in the region around the origin. Long runs down the x, y have not shown evidence for more planar maxima and minima (runs were made up to $x=\pm 700$, $y=\pm 700$). The fields tend towards zero far away down the axes. We have in Figure 1 placed a small box in regions where the magnitudes grow large. These regions we call "confluence" regions (Muraskin, 1980a). Here large negative numbers and large positive numbers are in close proximity.

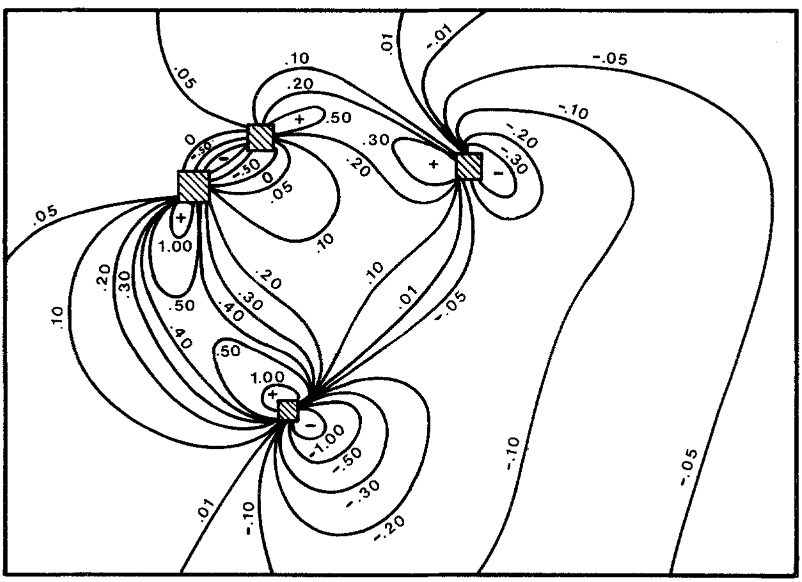

Fig. 1. Map for x and y of A^1_{11} around the origin showing four confluence regions (in boxes).

We next investigated maps for various values of z. We found different behavior with respect to the number of confluence regions. For example at $z=-9$ no confluence regions were seen. At $z=2.5$ there were two confluence regions. At $z=0$ there were four (Figure 1). At $z=9.5$ there were none. Thus, confluence regions were bounded from above and below in z. We followed the location of the confluence region for different values of z. We found that the confluence regions lie (in a course sense) on a closed curve (there was not much thickness to the confluence regions). A sketch of the confluence curve is given in Figure 2 for A^1_{11}. What we have is a loop. The bending of the loop allows for four confluence regions at above $z=0.5$.

We note that in Muraskin (1980a), in null theory, we saw the possibility for two confluence regions for each x, y map. The confluence region as a function of z was thought to give two lines (presuming that the second region can indeed be verified rather than just suggested). If the lines close into one another we would have a "topological" particle there as well. However, no evidence has been presented for such an effect.

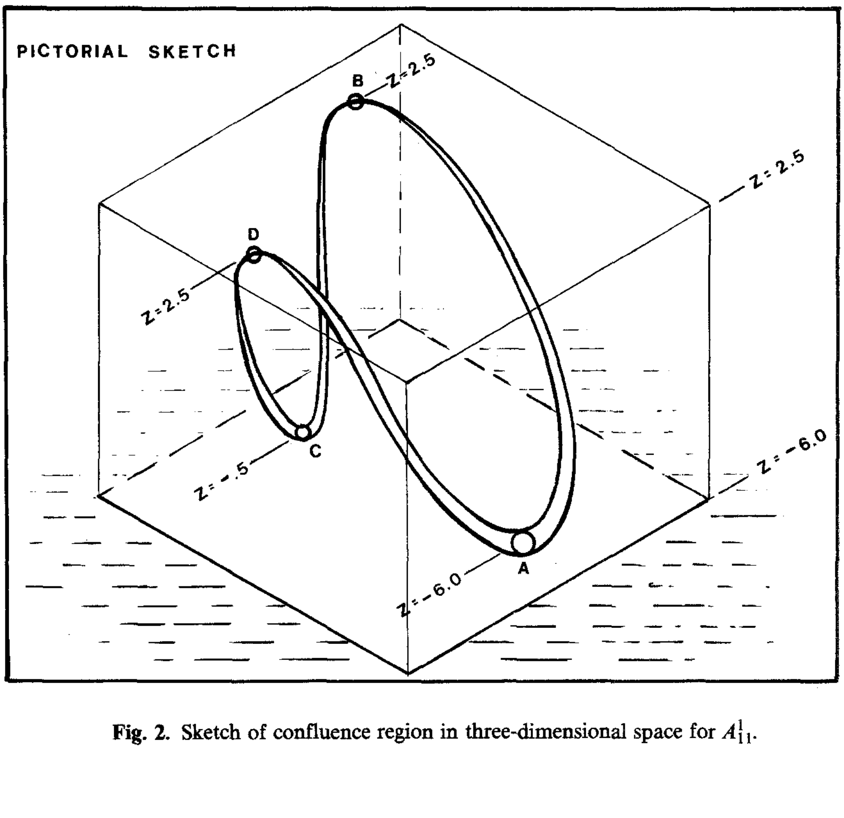

Fig. 2. Sketch of confluence region in three-dimensional space for A^1_{11}.

In this paper we have found that such a "topological" particle does exist after all in aesthetic field theory.

We next investigated the effect of the passage of time. We looked at maps at various instants in time. We found confluence regions for all instants that we studied. Confluence regions were found in the neighborhood of

$t=-19.5$	$t=-13.5$	$t=-6$	$t=6$
$z=10$	$z=5$	$z=0$	$z=-7$
$x=15.8$	$x=12.2$	$x=7.2$	$x=5.4$
$y=-3.8$	$y=-2.2$	$y=-4.8$	$y=-6.6$

The large magnitudes persisted as the time interval was varied.

We studied the particle in detail at $t=-6$. We again found the topological particle, although now the number of confluence regions for a particular z was seen at most to be 2. Spreading was not significant at $t=-6$. However, at $t=-19.5$ we found that the confluence region had a spread $\Delta z>22$ as contrasted with $\Delta z<8.5$ at $t=0$. It is not clear if the extension Δz will continue to increase as time goes on. We had a problem of greater extension also in Muraskin (1981). We did not attempt detailed pictures at higher times because of computer time limitations.

We still found for the different times the same effect that large magnitudes were tied to regions where negative numbers were in close proximity to positive numbers. Thus again we have the effect of confinement.

All field components have large magnitudes in the confluence region.

4. SUMMARY

A "topological" particle has been found within aesthetic field theory. It is not clear whether many such structures exist. Large negative and large positive numbers appear within the particle structure. It may be that large distances are a significant feature of the solutions as well. This would make the finding of many such "topological" particles difficult.

APPENDIX

To make the paper self contained we give a description of the origin point data used in this paper but introduced in Muraskin (1981). The set below of nonzero gammas is invariant under three dimensional rotations:

$$\Gamma^1_{10}=\Gamma^2_{20}=\Gamma^3_{30}=\Gamma^0_{00}=\Gamma^1_{01}=\Gamma^2_{02}=\Gamma^3_{03}=A$$

$$\Gamma^0_{11}=\Gamma^0_{22}=\Gamma^0_{33}=-B$$

$$\Gamma^2_{13}=\Gamma^3_{21}=\Gamma^1_{32}=-\Gamma^1_{23}=-\Gamma^3_{12}=-\Gamma^2_{31}=+C \tag{17}$$

Performing an inversion on the set (17) gives

$$\Gamma^1_{10}=\Gamma^2_{20}=\Gamma^3_{30}=\Gamma^0_{00}=\Gamma^1_{01}=\Gamma^2_{02}=\Gamma^3_{03}=A$$

$$\Gamma^0_{11}=\Gamma^0_{22}=\Gamma^0_{33}=-B$$

$$\Gamma^2_{13}=\Gamma^3_{21}=\Gamma^1_{32}=-\Gamma^1_{23}=-\Gamma^3_{12}=-\Gamma^2_{31}=-C \tag{18}$$

Both sets of data can be incorporated into the theory by taking (17) to be $\mathrm{Re}\,\Gamma^\alpha_{\beta\gamma}$ and (18) to be $\mathrm{Im}\,\Gamma^\alpha_{\beta\gamma}$. We work here with the case $A=B=C=0.1$. We use a complex $e^\alpha i$ as follows:

$$e^\alpha{}_i=f^\alpha{}_i+ih^\alpha{}_i \tag{19}$$

with

$f^1{}_1=0.88$	$f^1{}_2=-0.42$	$f^1{}_3=-0.32$	$f^1{}_0=0.2$	
$f^2{}_1=0.5$	$f^2{}_2=0.7$	$f^2{}_3=-0.425$	$f^2{}_0=0.3$	
$f^3{}_1=0.2$	$f^3{}_2=-0.55$	$f^3{}_3=0.89$	$f^3{}_0=0.6$	
$f^0{}_1=0.44$	$f^0{}_2=-0.16$	$f^0{}_3=0.39$	$f^0{}_0=1.01$	(20)
$h^1{}_1=0.3$	$h^1{}_2=-0.2$	$h^1{}_3=0.11$	$h^1{}_0=0.15$	
$h^2{}_1=-0.24$	$h^2{}_2=-0.16$	$h^2{}_3=0.09$	$h^2{}_0=0.07$	
$h^3{}_1=0.13$	$h^3{}_2=-0.26$	$h^3{}_3=0.31$	$h^3{}_0=-0.086$	
$h^0{}_1=0.05$	$h^0{}_2=0.1$	$h^0{}_3=-0.26$	$h^0{}_0=-0.31$	

Use was made of

$$\Gamma^i_{jk}=e_\alpha{}^i e^\beta{}_j e^\gamma{}_k \Gamma^\gamma{}_{\beta\gamma} \tag{21}$$

This set of origin point data leads to a "topological" particle. It is not clear that this is the only set of data which leads to this type of result.

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